DESIGN OF MACHINE ELEMENTS

Module-II

Nivish George

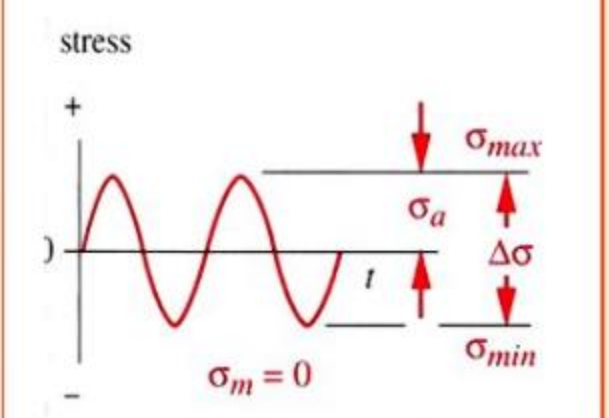
RSET
RAJAGIRI SCHOOL OF ENGINEERING & TECHNOLOGY

Fluctuating Stresses

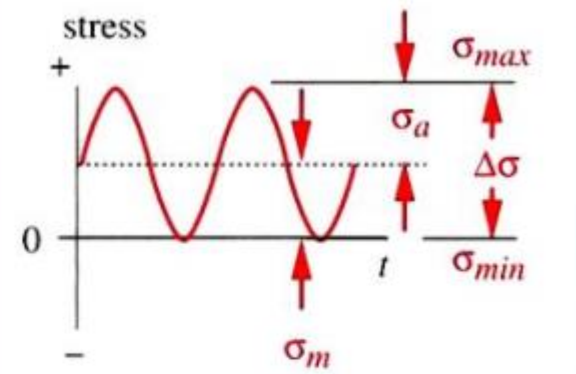
Stress that arise due to the variation in magnitude of force with respect to time

$$\sigma_m = \frac{1}{2}(\sigma_{max} + \sigma_{min})$$

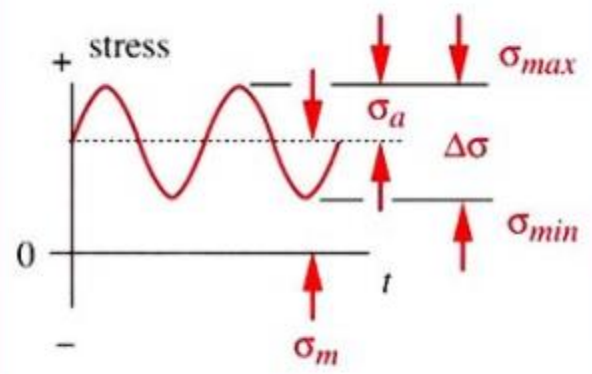
$$\sigma_a = \frac{1}{2}(\sigma_{max} - \sigma_{min})$$



(a) Fully reversed



(b) Repeated



(c) Fluctuating

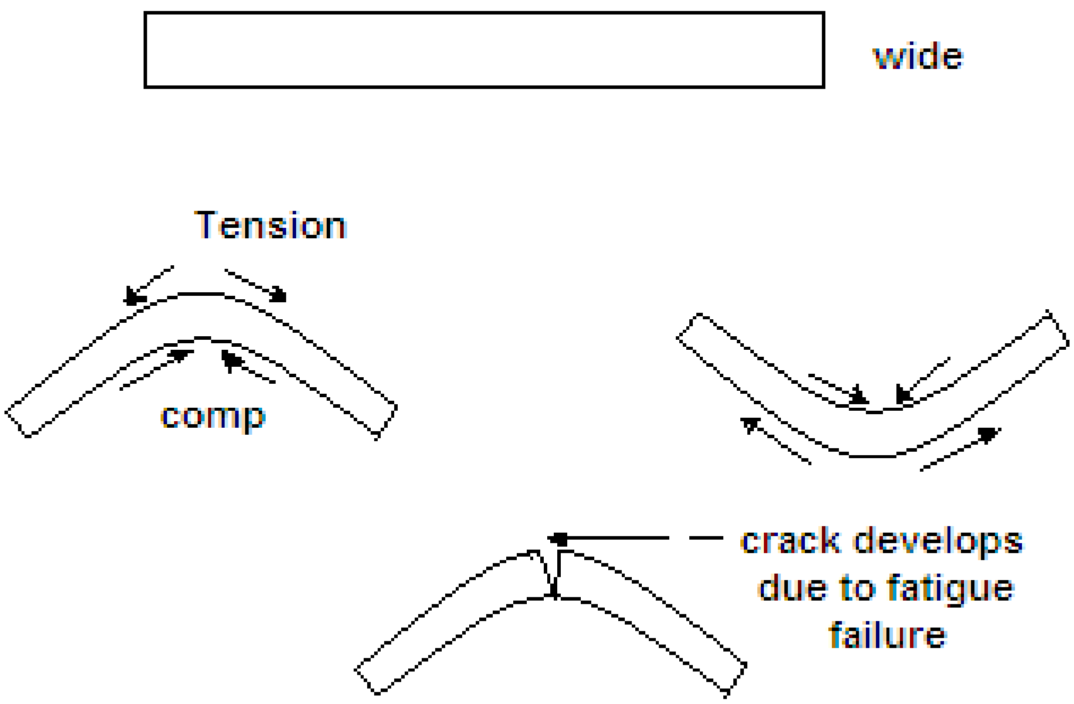
Fatigue Failure

Failure < Ultimate tensile strength of the material

Time delayed fracture under cyclic loading: Fatigue failure

<u>Common examples</u>

- Transmission shafts,
- connecting rods,
- gears,
- suspension springs,
- ball bearings



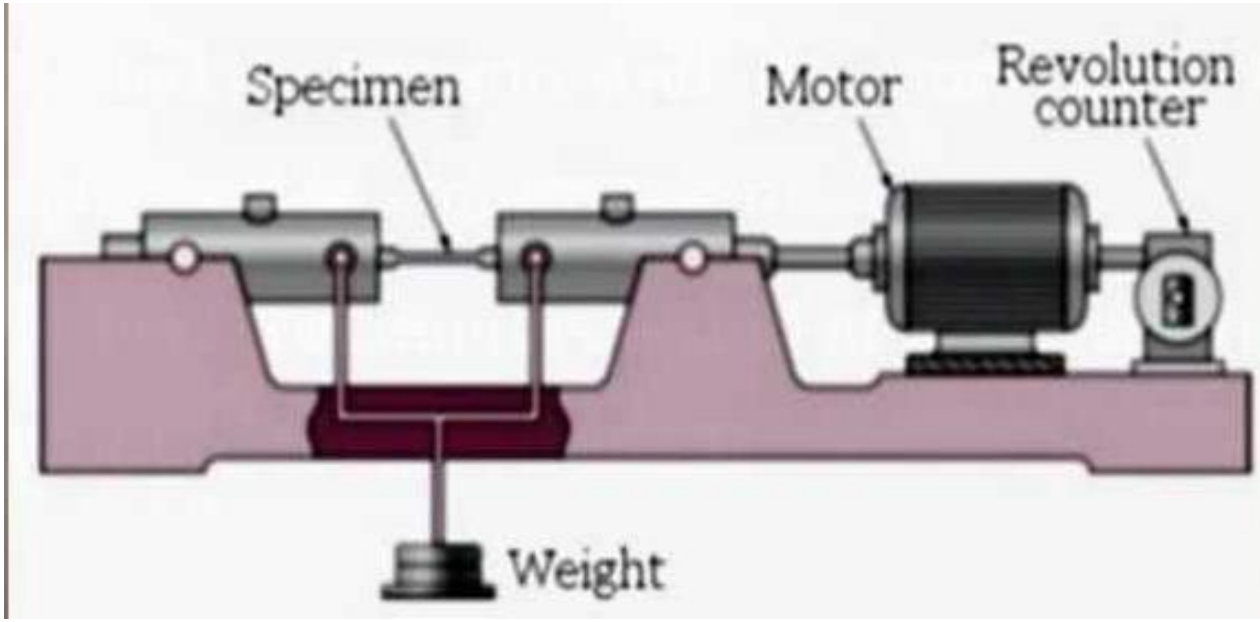
Fatigue Failure Regions

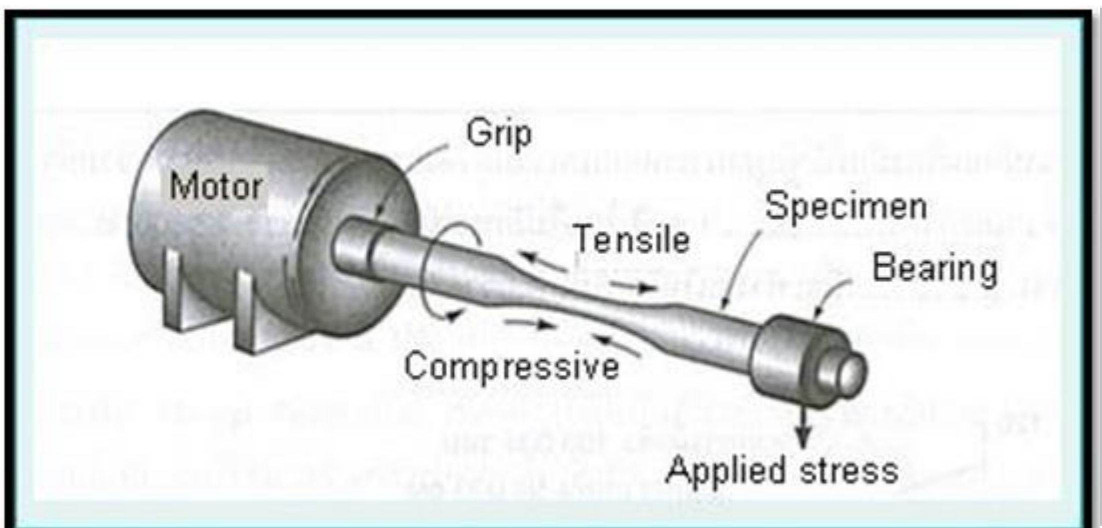
Endurance limit

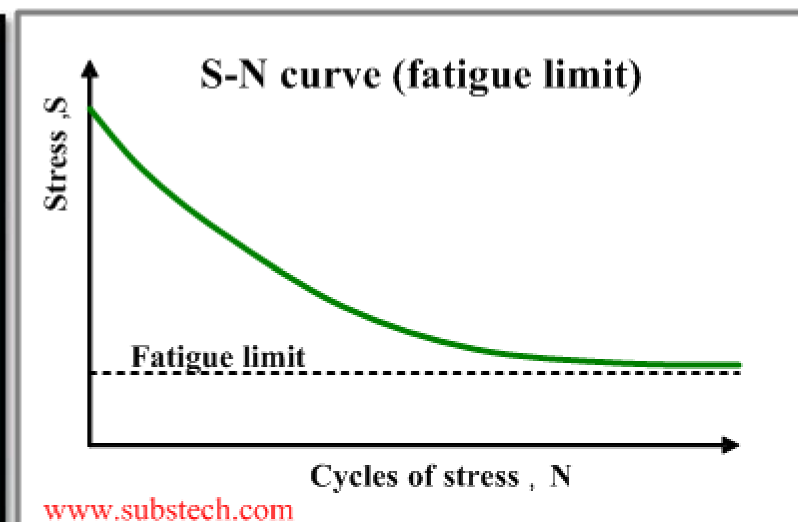
Endurance limit is the maximum amplitude of completely reversed stress that the standard specimen can sustain for an unlimited number of cycles without fatigue failure

Fatigue life is defined as the number of stress cycles that the standard specimen can complete during the test before the appearance of the first fatigue crack

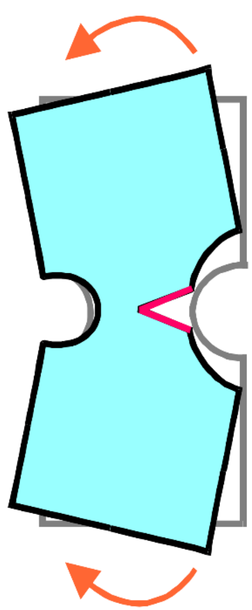
Fatigue Testing





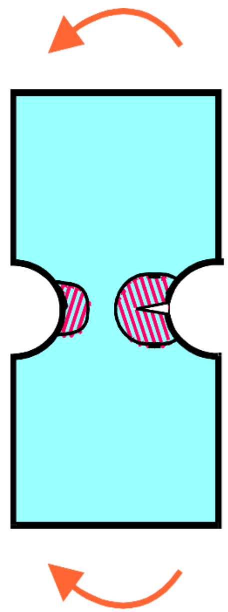


Low Cycle and High Cycle Fatigue



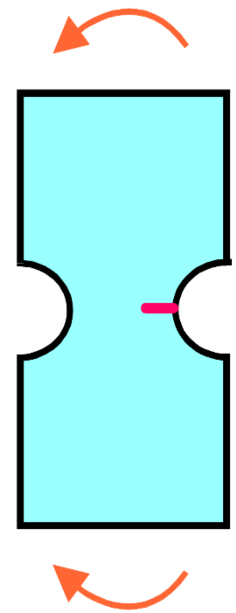
Static failure
(large global deformation and large plastic strain)

0 cycle



Low cycle fatigue
(small local plastic strains)

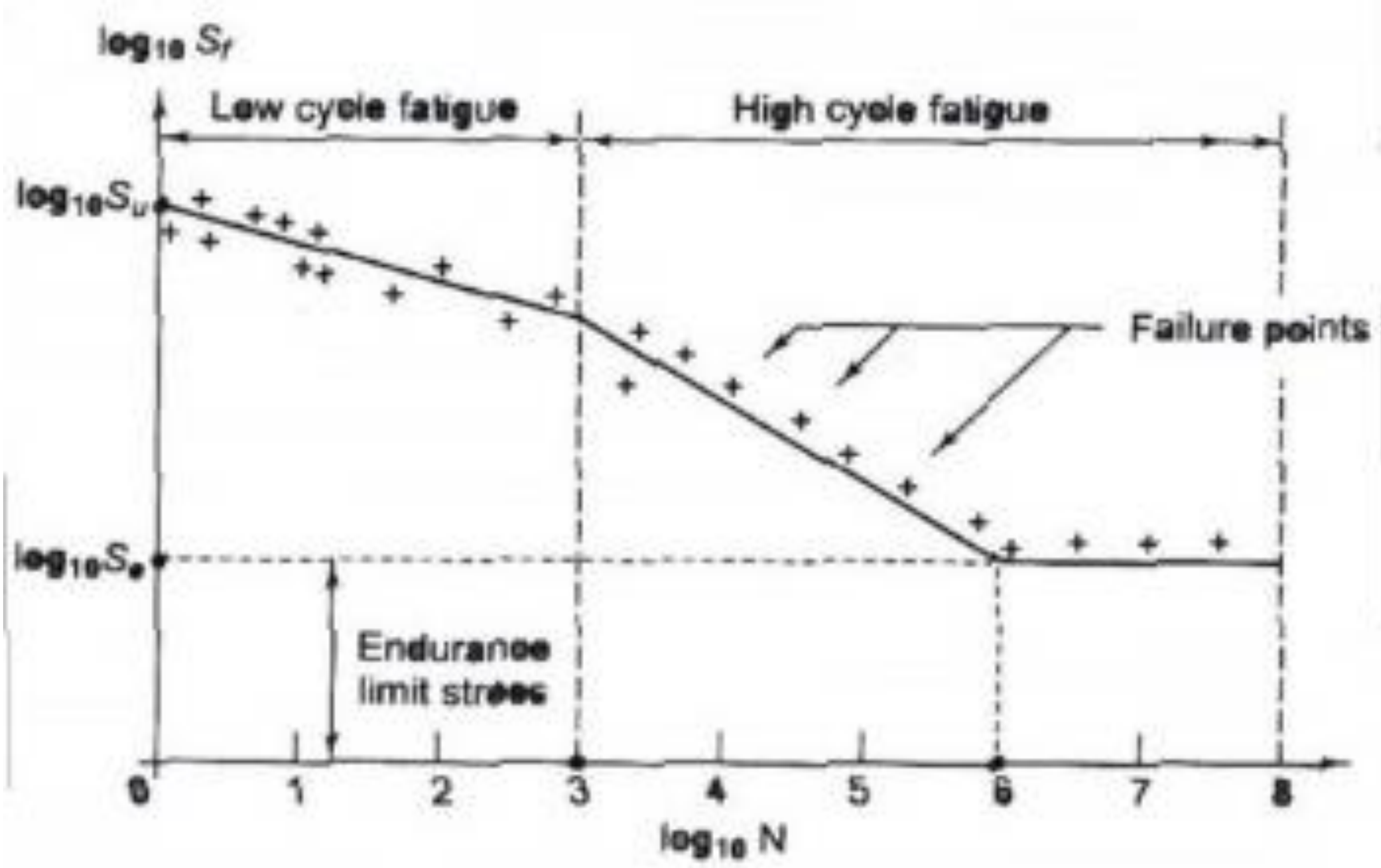
0-10^3 cycles



High cycle fatigue
(small local elastic strains)

10^3 -10^8 cycles

Low Cycle and High Cycle Fatigue



Notch Sensitivity

$$K_{tf} = \frac{Endurance\ limit\ of\ the\ notch\ free\ specimen}{Endurance\ limit\ of\ the\ notched\ specimen}$$

Eqn 2.12(a)

Notch sensitivity is defined as the susceptibility of a material to succumb to the damaging effects of stress raising notches in fatigue loading.

$$q = \frac{Increase\ of\ actual\ stress\ over\ nominal\ stress}{Increase\ of\ theoretical\ stress\ over\ nominal\ stress}$$

$$q = \frac{(K_{tf}\sigma_o - \sigma_o)}{(K_t\sigma_o - \sigma_o)}$$

Eqn 2.4

$$K_{tf} = 1 + q(K_t - 1)$$

Eqn 2.5

Endurance limit: Estimation

$$S_e = K_a K_b K_c K_d S'_e$$

- S_e: endurance limit of a particular mechanical component subjected to reversed bending stress
- S'_e: endurance limit stress of a rotating beam specimen subjected to reversed bending stress
- K_a: Surface finish factor
- K_b: Size factor
- K_c: Reliability factor
- K_d: Modifying factor accounting stress concentration factor

Surface finish factor (K_a)

- K_a: To account for the stress raisers due to the poor surface finish
- They are derating factors
- $K_a = a(S_{ut})^b$ (shigley and Mischke)

Coefficient values for steel

Surface finish	a	b
Ground	1.58	-0.085
Machined or cold drawn	4.51	-0.265
Hot-rolled	57.7	-0.718
As forged	272	-0.995

Size factor (K_b)

- K_b: To account for the increase in size
- For 2.79 mm ≤ d ≤ 51 mm
  - $K_b = 1.24(d)^{-0.107}$ (shigley and Mischke)
- For 51 mm ≤ d ≤ 254mm
  - $K_b = 0.859 - 0.000873d$

Values of size factor

Diameter (d) mm	K_b
d ≤ 7.5	1
7.5 ≤ d ≤ 50	0.85
d > 50	0.75

Reliability Factor (K_c)

- K_c: To account for the dispersion of data obtained from the experimental tests
- Standard deviation of test is 8% of mean value
- K_c=1, Reliability =50%

Reliability factor

Reliability R (%)	K_c
50	1.000
90	0.897
95	0.868
99	0.814
99.9	0.753
99.99	0.702

Modifyng Factor accounting stress concentration (K_d)

- K_d: To account for the stress concentration
- $K_d = \frac{1}{K_f}$

Axial loading to rotating beam bending

$$(S_e)_a = 0.8\, S_e$$

Design for finite and infinite life (Fully reversed cycle)

Case 1: Design for infinite life

Endurance limit becomes the failure criterion

$$\sigma_a = \frac{S_e}{fs} \qquad\qquad \tau_a = \frac{S_{se}}{fs}$$

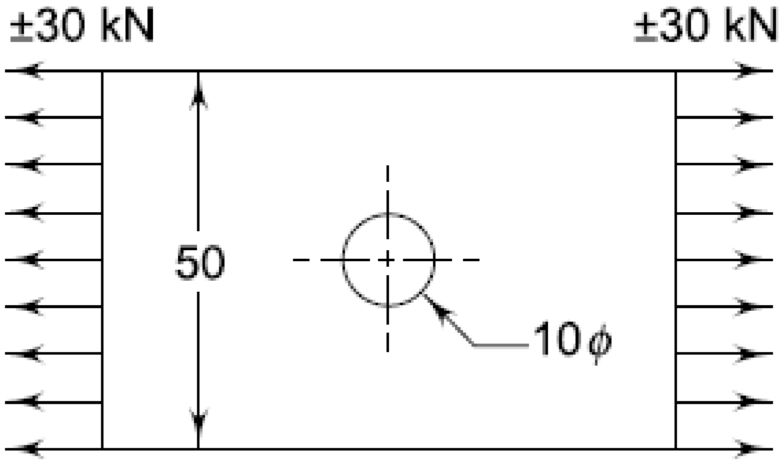
Case 2: Design for finite life

A straight line AB drawn from ($0.9\ S_{ut}$) at 10^3 cycle to (S_e) at 10^6 cycles on a log-log paper

- Locate point A and B
- Join AB
- Depending on life N obtain S_f

Problem 2.1

A plate made of steel 20C8 (S_{ut}=440 N/mm^2) is hot rolled and normalised condition is shown in Figure. It is subjected to a completely reversed axial load of 30 kN. The notch sensitivity factor q can be taken as 0.8 and the expected reliability is 90%. The size facto is 0.85. The factor of safety is 2. Determine the plate thickness for infinite life

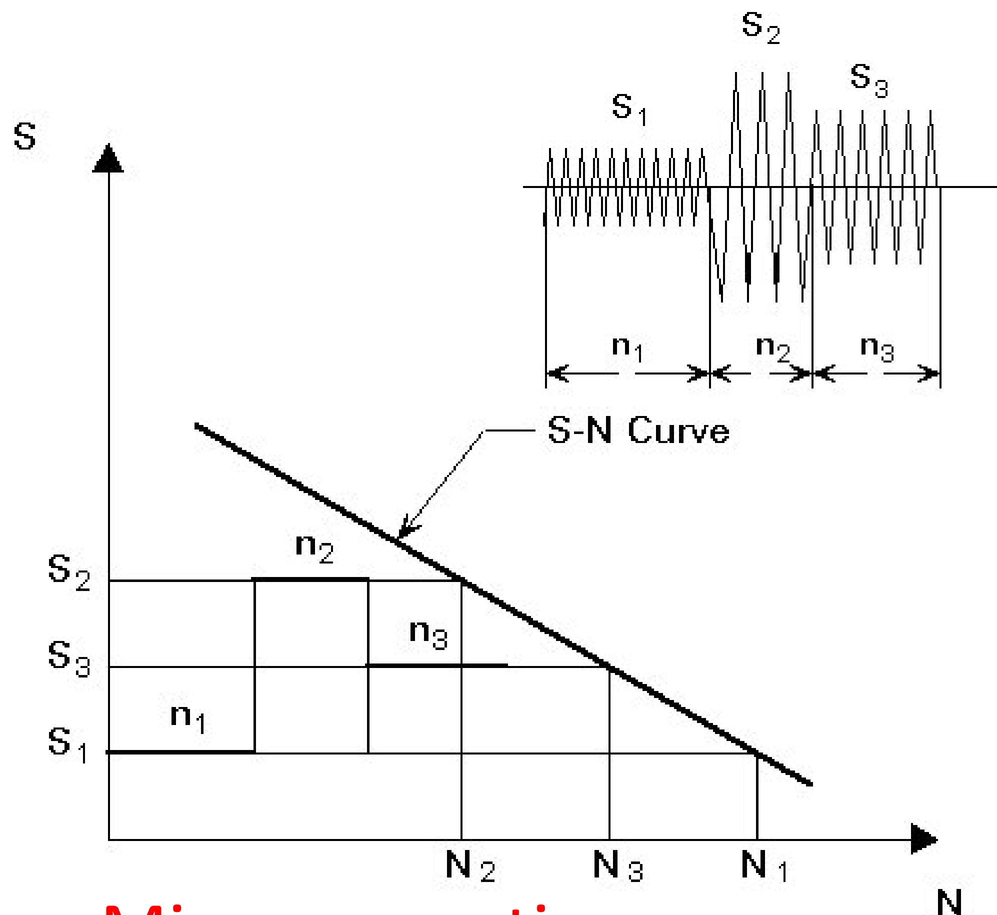


Cumulative damage in fatigue

First set of cycle = n_1
Stress in the first cycle = S_1
Fatigue life with n_1 cycle = N_1

Second set of cycle = n_2
Stress in the second cycle = S_2
Fatigue life with n_2 cycle = N_2

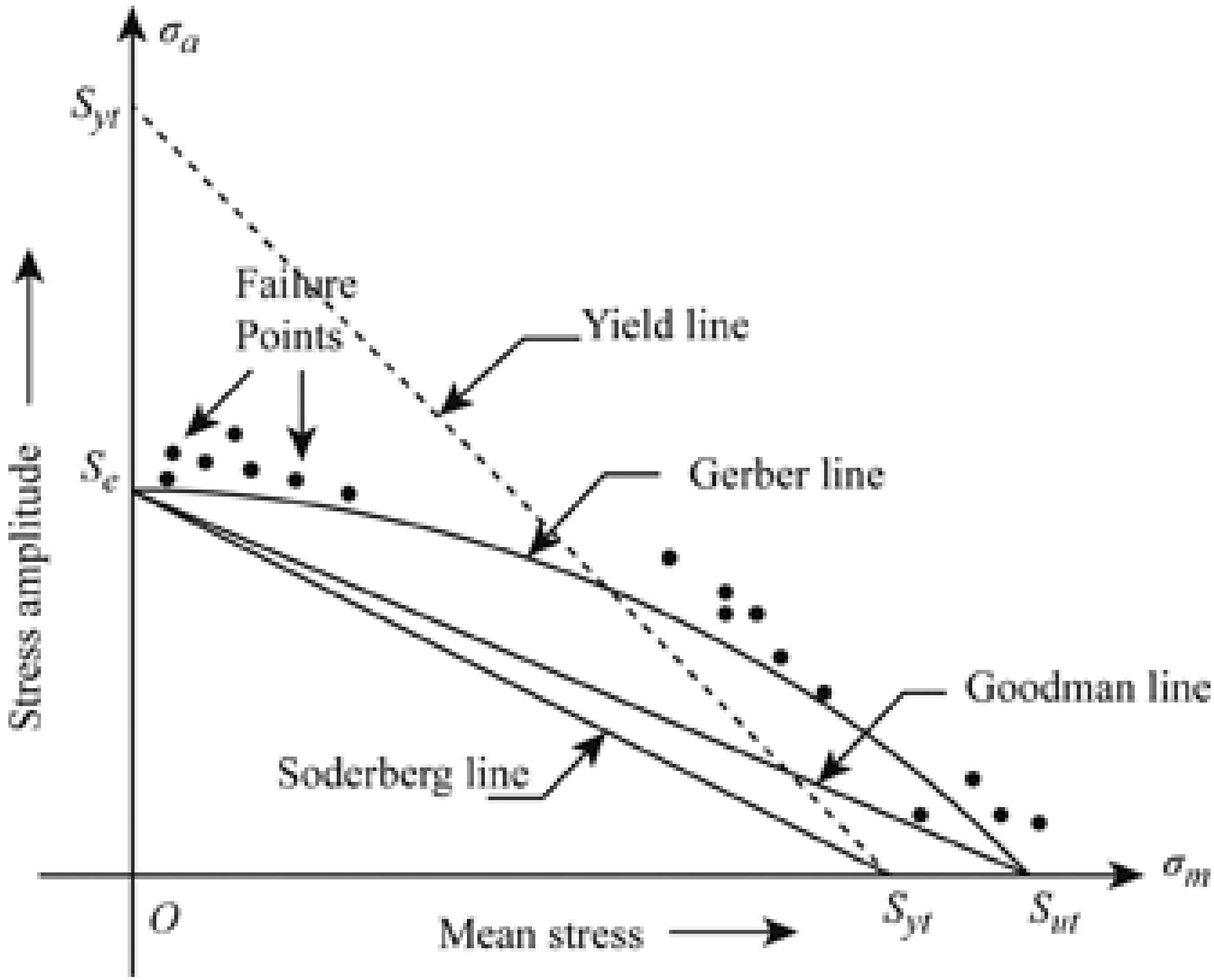
x^{th} set of cycle = n_x
Stress in the x^{th} set = S_x
Fatigue life with n_x cycle = N_x



Miners equation

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \; \dots \dots . + \frac{n_x}{N_x} = 1$$

Soderberg, Goodman and Gerber line



Soderberg, Goodman and Gerber line

Gerber line: A parabolic curve joining S_e on the ordinate to S_{ut} on the abscissa

$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$

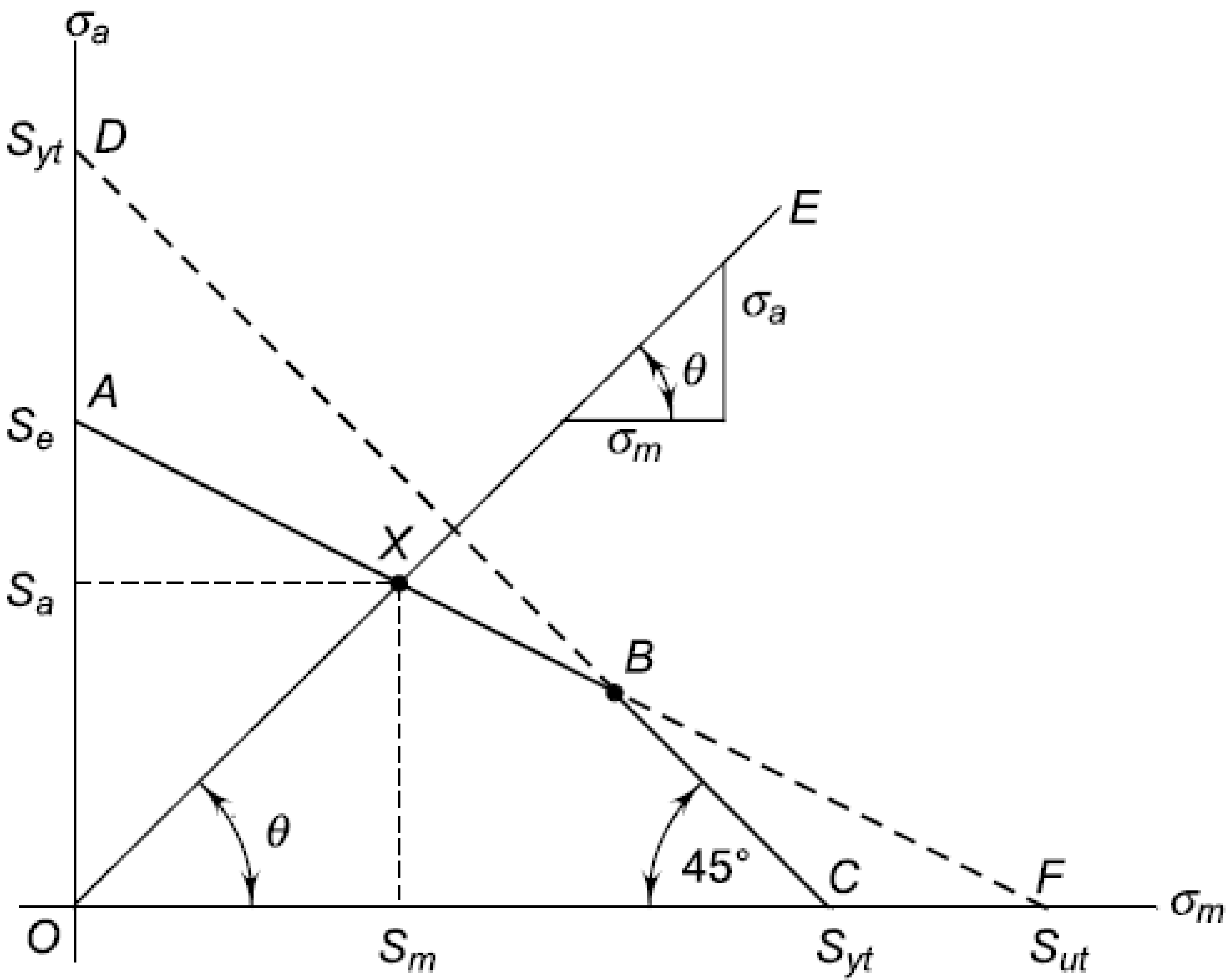
Soderberg line: A straight line joining S_e on the ordinate to S_{yt} on the abscissa

$$\frac{S_m}{S_{yt}} + \frac{S_a}{S_e} = 1$$

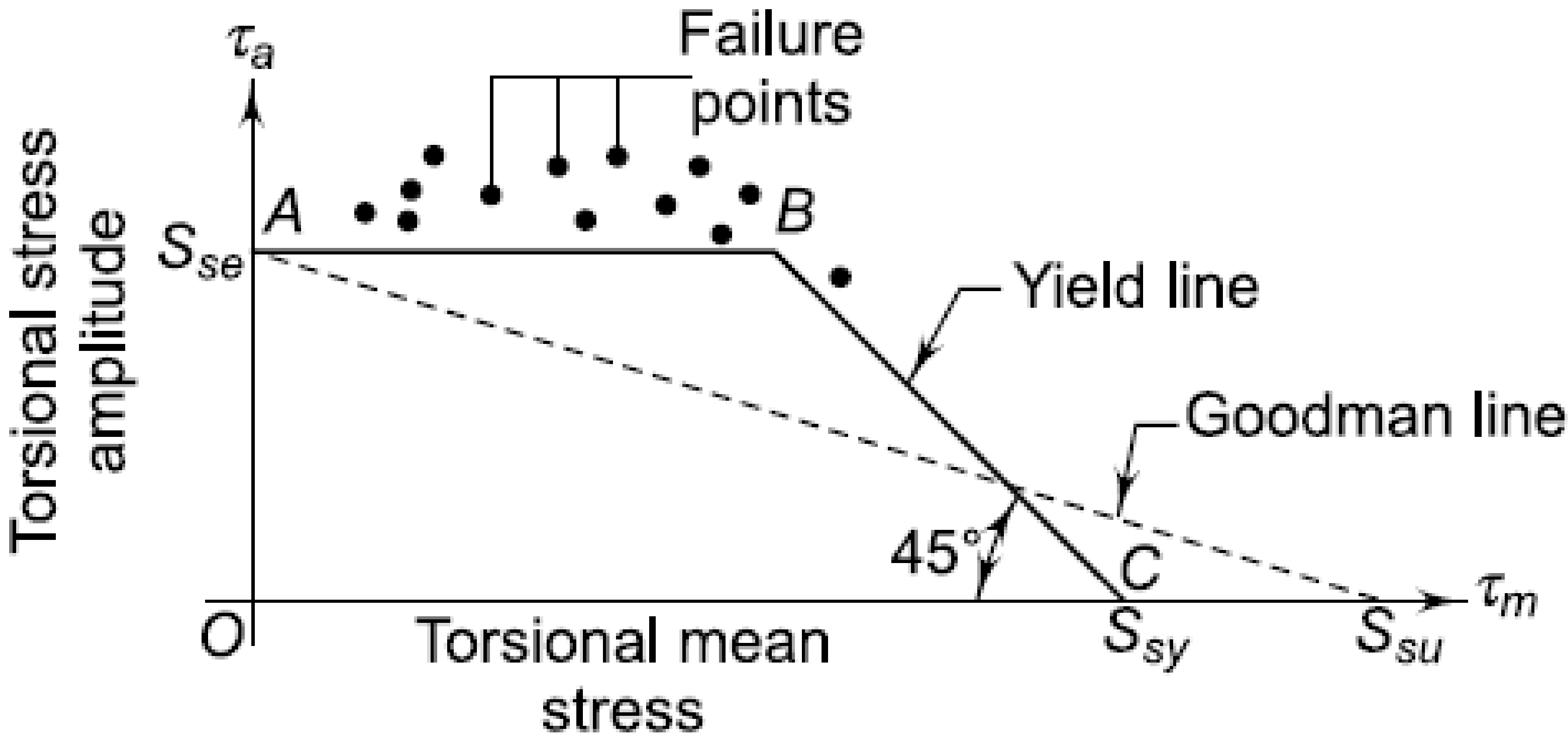
Goodman line: A straight line joining S_e on the ordinate to S_{ut} on the abscissa

$$\frac{S_m}{S_{ut}} + \frac{S_a}{S_e} = 1$$

Modified Goodman Diagrams: Pure Bending or Tensile



Modified Goodman Diagrams: Pure Torsional Load



Fatigue Design Under Combined Loading

Most general equation of distortion energy theory

$$\sigma^2 = \frac{1}{2}\left[\left((\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\right)\right]$$

For a 2D normal stress case

$$\sigma_m = \sqrt{(\sigma_{xm}^2 - \sigma_{xm}\sigma_{ym} + \sigma_{ym}^2)}$$

$$\sigma_a = \sqrt{(\sigma_{xa}^2 - \sigma_{xa}\sigma_{ya} + \sigma_{ya}^2)}$$

Combined bending and torsional case

$$\sigma_m = \sqrt{(\sigma_{xm}^2 + 3\tau_{xym}^2)}$$

$$\sigma_a = \sqrt{(\sigma_{xa}^2 + 3\tau_{xya}^2)}$$

Problem

A transmission shaft carries a pulley midway between the two bearings. The bending moment at the pulley varies from 200 N-m to 600 N-m, as the torsional moment in the shaft varies from 70 N-m to 200 N-m. The frequencies of variation of bending and torsional moments are equal to the shaft speed: The shaft is made of steel FeE400 (S_{ut} = 540 N/mm^2 and S_{yt} = 400 N/mm^2). The corrected endurance limit of the shaft is 200 N/mm^2. Determine the diameter of the shaft using a factor of safety of 2.

Impact Stresses

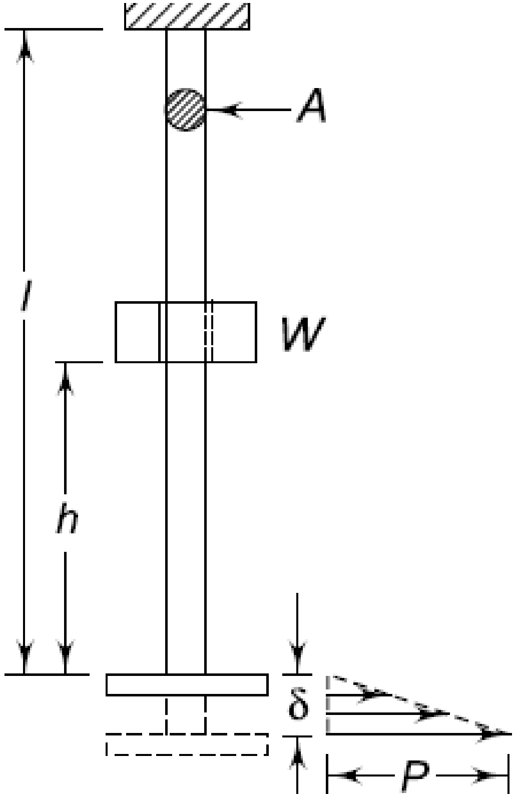
Defined as a collision of one component in motion with a second component, which may be either in motion or at rest

Load which is applied rapidly to the machine component

- Energy released (weight)= W(h+δ)
- Energy absorbed (load × Deflection)= $\frac{1}{2}P\delta$

Energy absorbed = Energy released

$$\frac{P}{W} = \left[1 + \sqrt{1 + \frac{2hAE}{Wl}}\right]$$



Factor of safety

Reserve strength:

$$fs = \frac{failure\ stress}{allowable\ stress} = \frac{failure\ load}{working\ load}$$

- For ductile materials
  - $fs = \frac{S_{yt}}{\sigma}$

- For brittle materials
  - $fs = \frac{S_{ut}}{\sigma}$

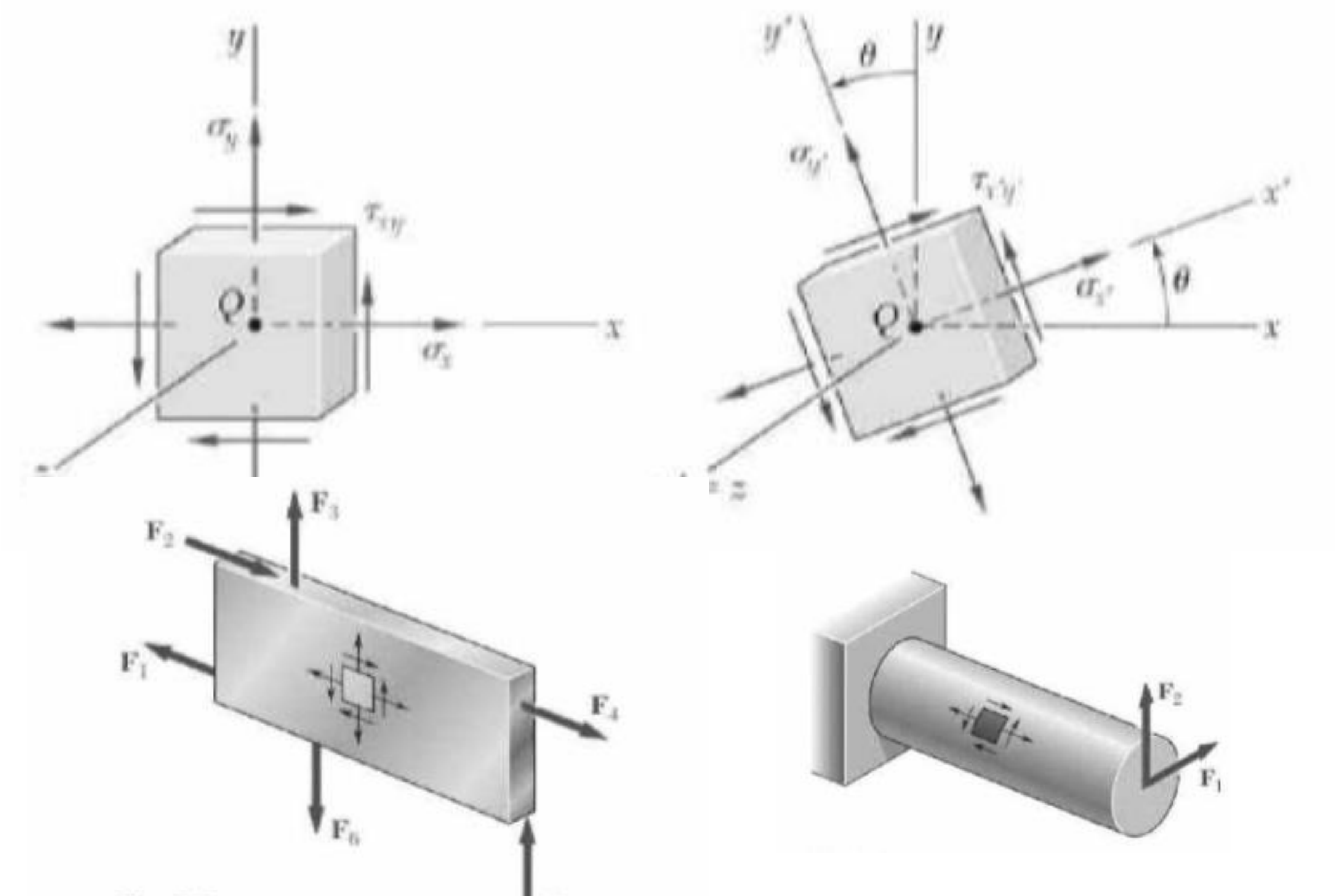
Reason for Factor of safety

- Uncertainty in magnitude of external force
- Variation in properties
- Variation in dimensions of the component
- Assumptions of material properties

Factor of safety: Magnitude Selection

- Effect of failure
- Type of load
- Material of component
- Degree of accuracy in force analysis
- Reliability of the component
- Cost of the component
- Testing of machine element
- Service conditions
- Quality of manufacture

Stress state



Mohr's Circle

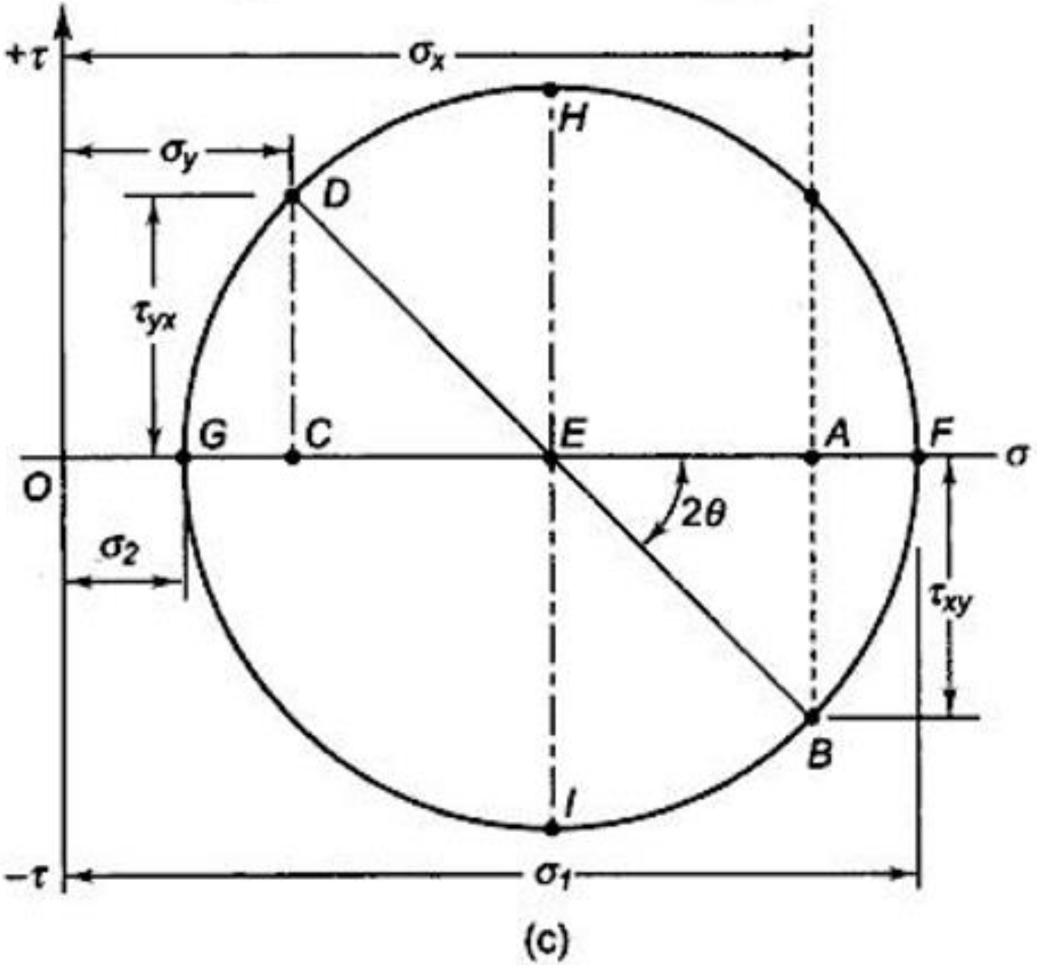
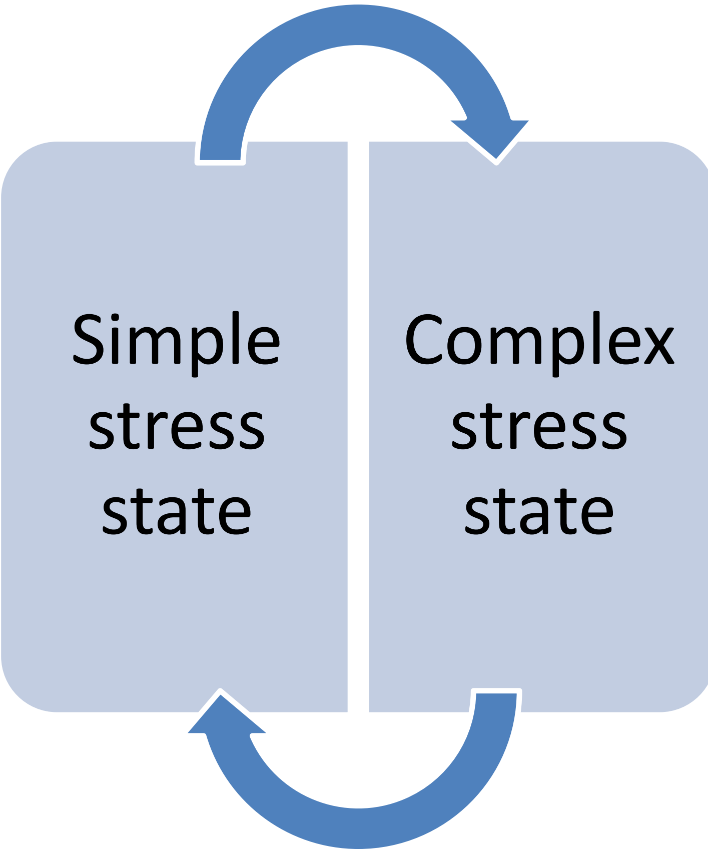


Fig. 4.30 *(a) Two-Dimensional State of Stress (b) Stresses on Oblique Plane (c) Mohr's Circle Diagram*

Principal Stress

Principal shear stress????

Theories of Elastic failure



- Maximum principal stress theory
  - (Rankine's theory)
- Maximum shear stress theory
  - (Coulumb, Tresca and Guest's theory)
- Distortion energy theory
  - (Huber von mises and Hencky's theory)
- Maximum strain theory
  - (St. Venant's theory)
- Maximum total strain energy theory
  - (Haigh's theory)

Maximum principal stress theory (Rankine's theory)

The failure of mechanical component subjected to bi-axial or tri-axial stresses occurs when the maximum principal stress reaches the yield or ultimate strength of the material.

If, $\sigma_1 > \sigma_2 > \sigma_3$

$$\sigma_1 = S_{yt} \text{ or } \sigma_1 = S_{ut}$$

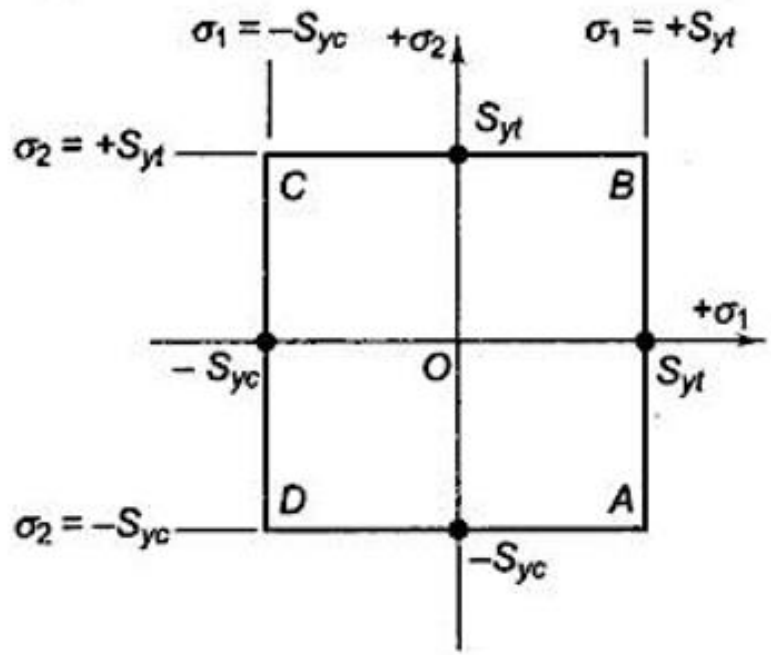


Fig. 4.31 *Boundary for Maximum Principal Stress Theory under Bi-axial Stresses*

Maximum shear stress theory (Guest's theory)

The failure of mechanical component subjected to bi-axial or tri-axial stresses occurs when the maximum shear stress at any point in the component becomes equal to the maximum shear stress in the standard specimen of the tension test, when yielding starts

$$\tau_{max} = \frac{\sigma_1}{2} = \frac{S_{yt}}{2}$$

Yield strength in shear is half of the yield strength in tension

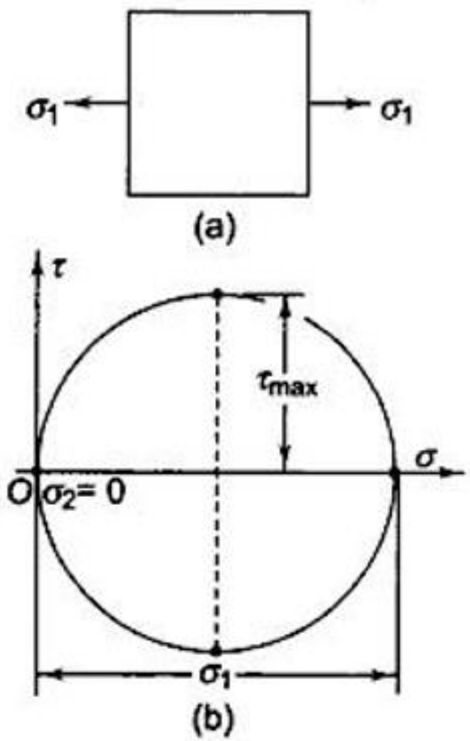
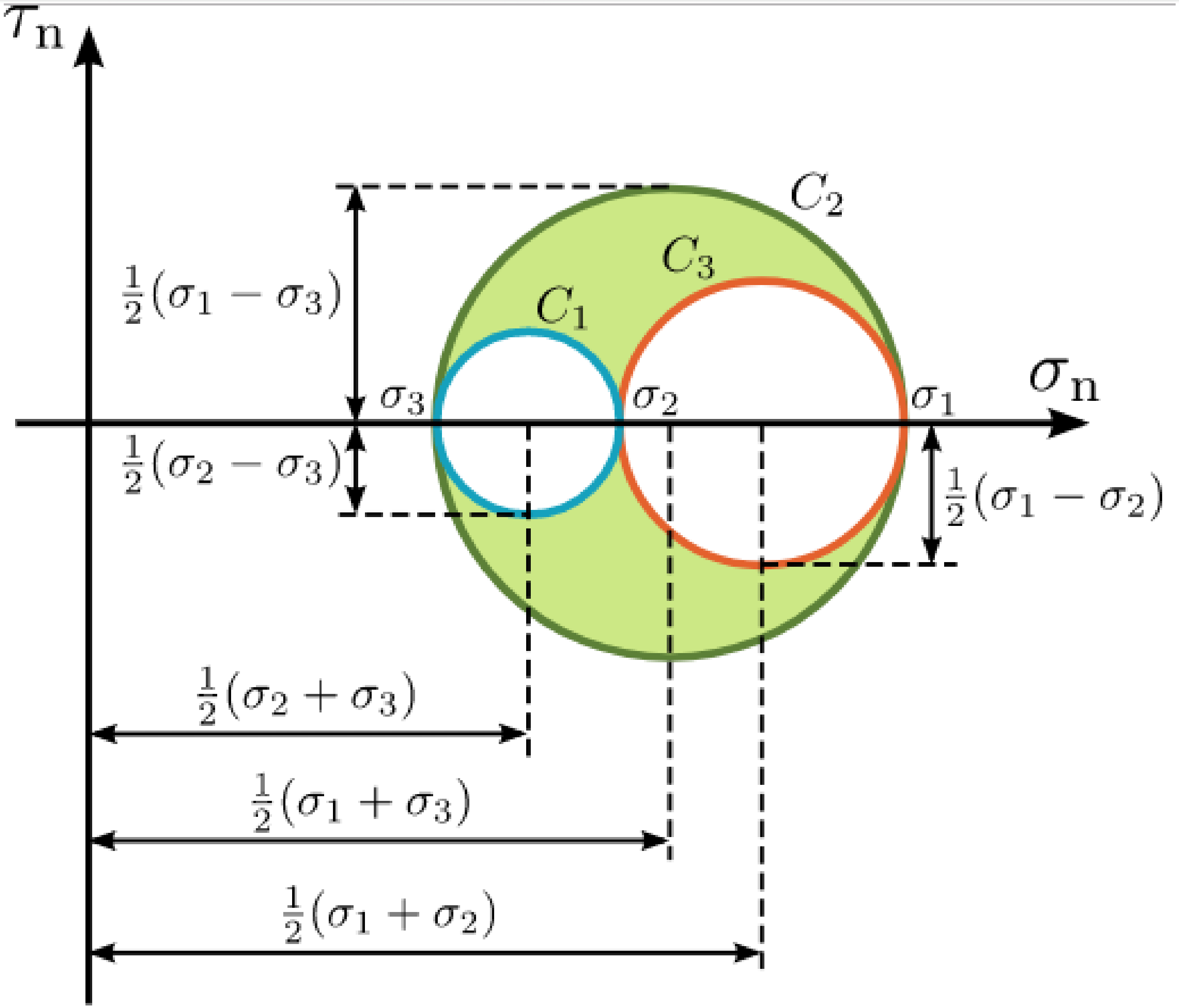


Fig. 4.32 *(a) Stresses in Simple Tension Test*
(b) Mohr's Circle for Stresses

Mohr's diagram



Maximum shear stress theory (Guest's theory)

$$\tau_{12} = \frac{\sigma_1 - \sigma_2}{2} = \frac{S_{yt}}{2}$$
$$\tau_{23} = \frac{\sigma_2 - \sigma_3}{2} = \frac{S_{yt}}{2}$$
$$\tau_{31} = \frac{\sigma_3 - \sigma_1}{2} = \frac{S_{yt}}{2}$$

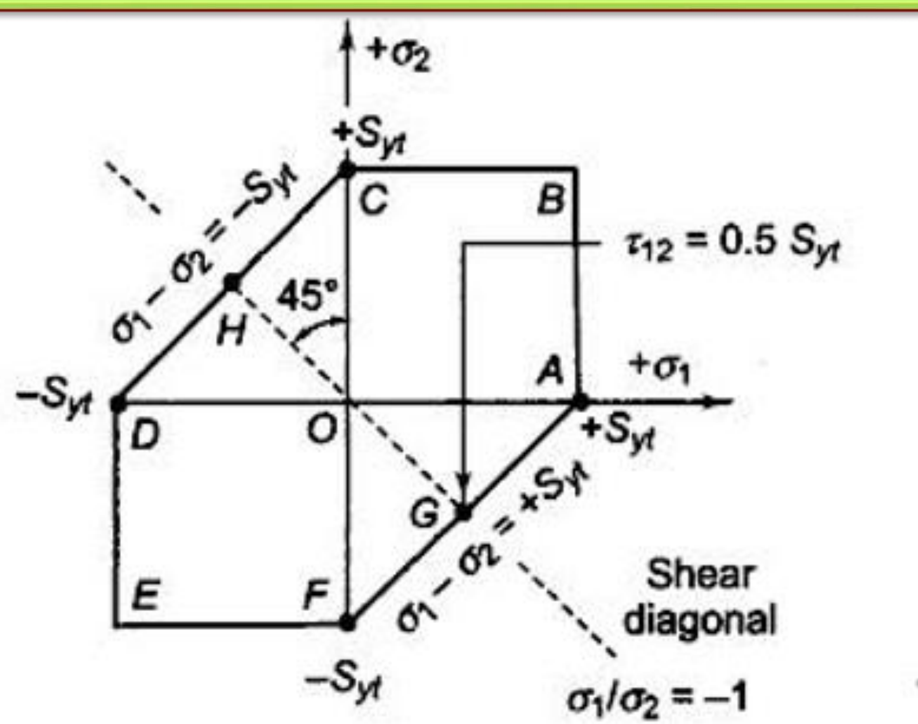


Fig. 4.33 *Boundary for Maximum Shear Stress Theory under Bi-axial Stresses*

$$\sigma_1 - \sigma_2 = S_{yt}$$
$$\sigma_2 - \sigma_3 = S_{yt}$$
$$\sigma_3 - \sigma_1 = S_{yt}$$

$$\sigma_1 - \sigma_2 = \pm S_{yt}$$
$$\sigma_2 = \pm S_{yt}$$
$$\sigma_1 = \pm S_{yt}$$

Distortion energy theory (Von Mises and Hencky's)

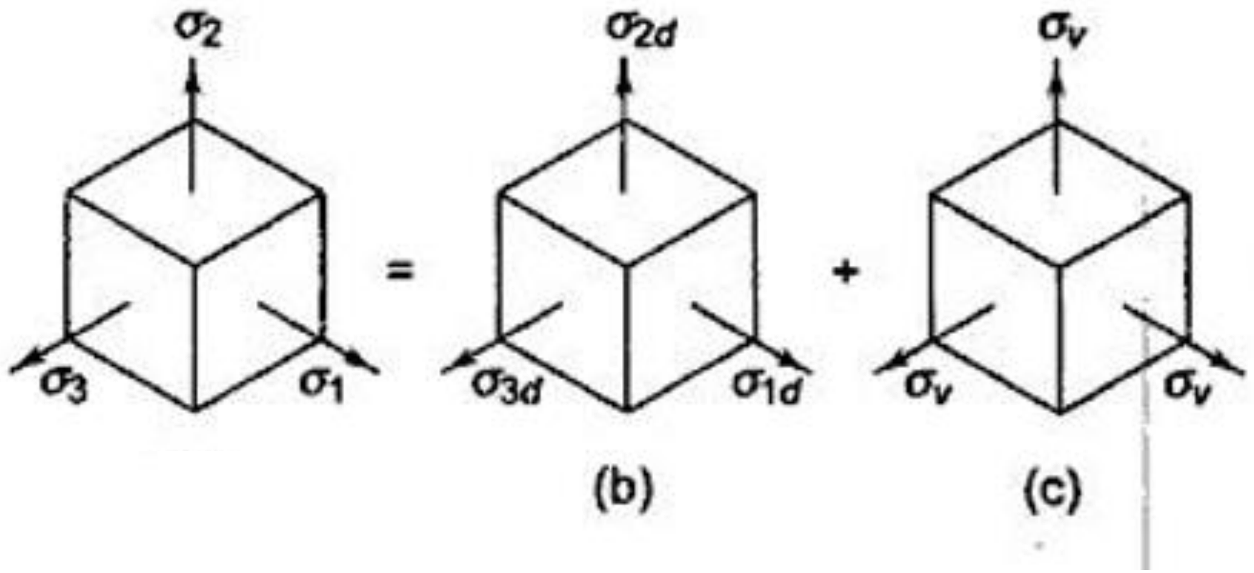
The theory states that the failure of mechanical component subjected to bi-axial and tri-axial stresses occurs when the strain energy of distortion per unit volume at any point in the component, becomes equal to the strain energy of distortion per unit volume in the standard specimen of tension-test, when yielding starts.

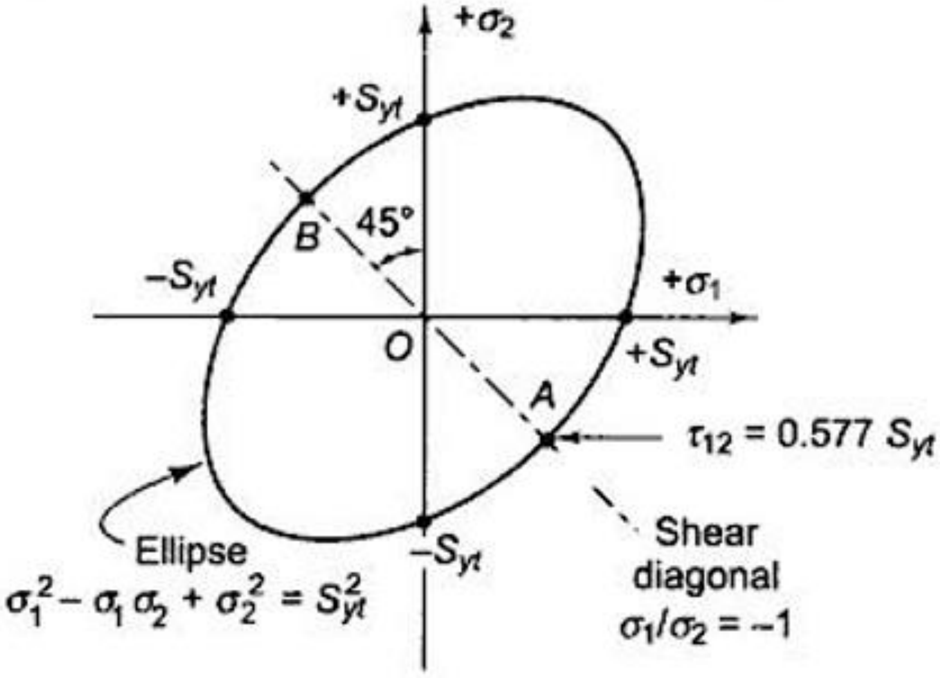
Total strain energy of the cube, U

$$U = \frac{1}{2}\sigma_1\epsilon_1 + \frac{1}{2}\sigma_2\epsilon_2 + \frac{1}{2}\sigma_3\epsilon_3$$

$$U = \frac{1}{2E}[(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\,]$$

Distortion energy theory (Von Mises and Hencky's)





$$S_{yt}^2 = \sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2$$
$$a^2 = x^2 - xy + y^2$$

Comparison

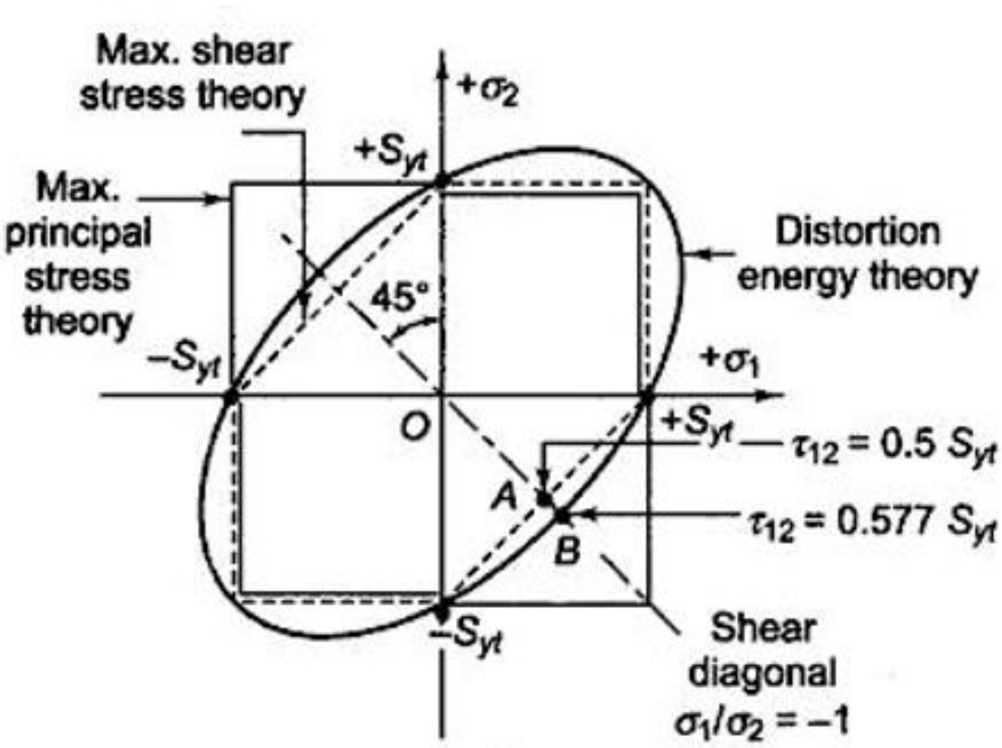


Fig. 4.37 *Comparison of Theories of Failure*

Ductile materials

Von Mises and Guest's theory

Brittle materials

Rankine's theory

Von Mises

Precise determination on all quadrants

Close limit determination

Guest theory is on conservative side

Rankine sometimes unsafe

Maximum Principal Strain theory (St. Venant's theory)

The theory states that failure or yielding occurs at point in a member when maximum principal (or normal) strain in a bi-axial or tri-axial stress system reaches the limiting value of strain as determined from simple tension test

$$\varepsilon_{max} = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

$$\varepsilon_{max} = \frac{S_{yt}}{E}$$

$$S_{yt} = \sigma_1 - \mu\sigma_2$$

**Not commonly used*

Maximum Strain Energy theory (Haigh's theory)

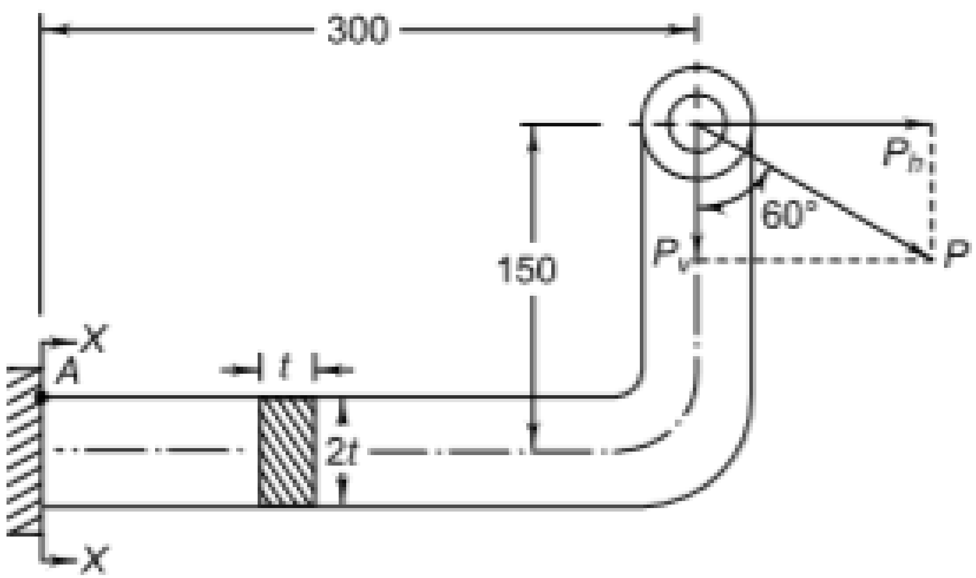
The theory states that failure or yielding occurs at point in a member when the strain energy per unit volume in a bi-axial or tri-axial stress system reaches the limiting strain energy as determined from simple tension test

$$U_1 = \frac{1}{2E}[(\sigma_1)^2 + (\sigma_2)^2 - 2\mu\sigma_1\sigma_2] \qquad U_2 = \frac{1}{2E}{S_{yt}}^2$$

$$(\sigma_1)^2 + (\sigma_2)^2 - 2\mu\sigma_1\sigma_2 = {S_{yt}}^2$$

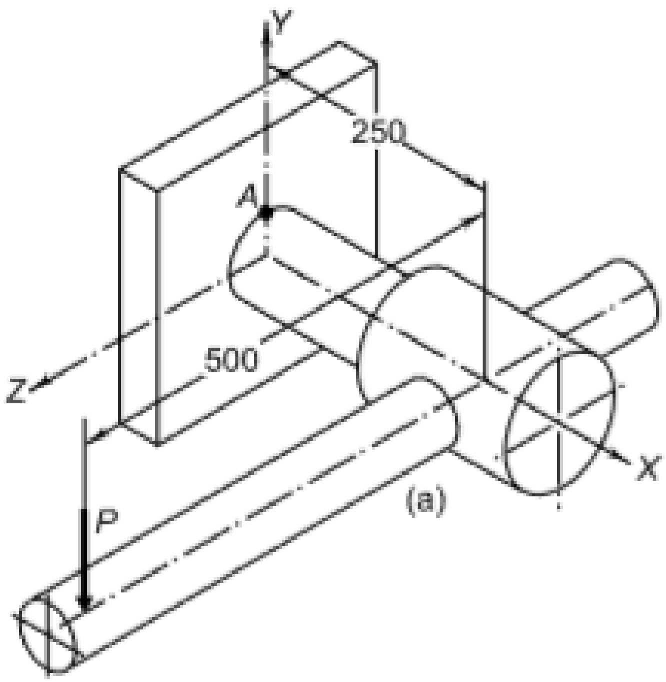
Problem 2.2

A wall bracket with a rectangular cross section is shown in Figure. The depth of the cross section is twice of the width. The force P acting on the bracket at 60^0 to the vertical is 5 kN. The material of the bracket is grey cast iron FG 200 and the factor of safety is 3.5. Determine the dimensions of the cross-section of the bracket. Assume maximum normal stress theory.



Problem 2.3

The shaft of an overhang crank subjected to a force P of 1 kN is shown in Figure. The shaft is made of plain carbon steel 45C8 and the tensile yield strength is 380 N/mm^2. The factor of safety is 2. Determine the diameter of the shaft using the maximum shear stress theory.

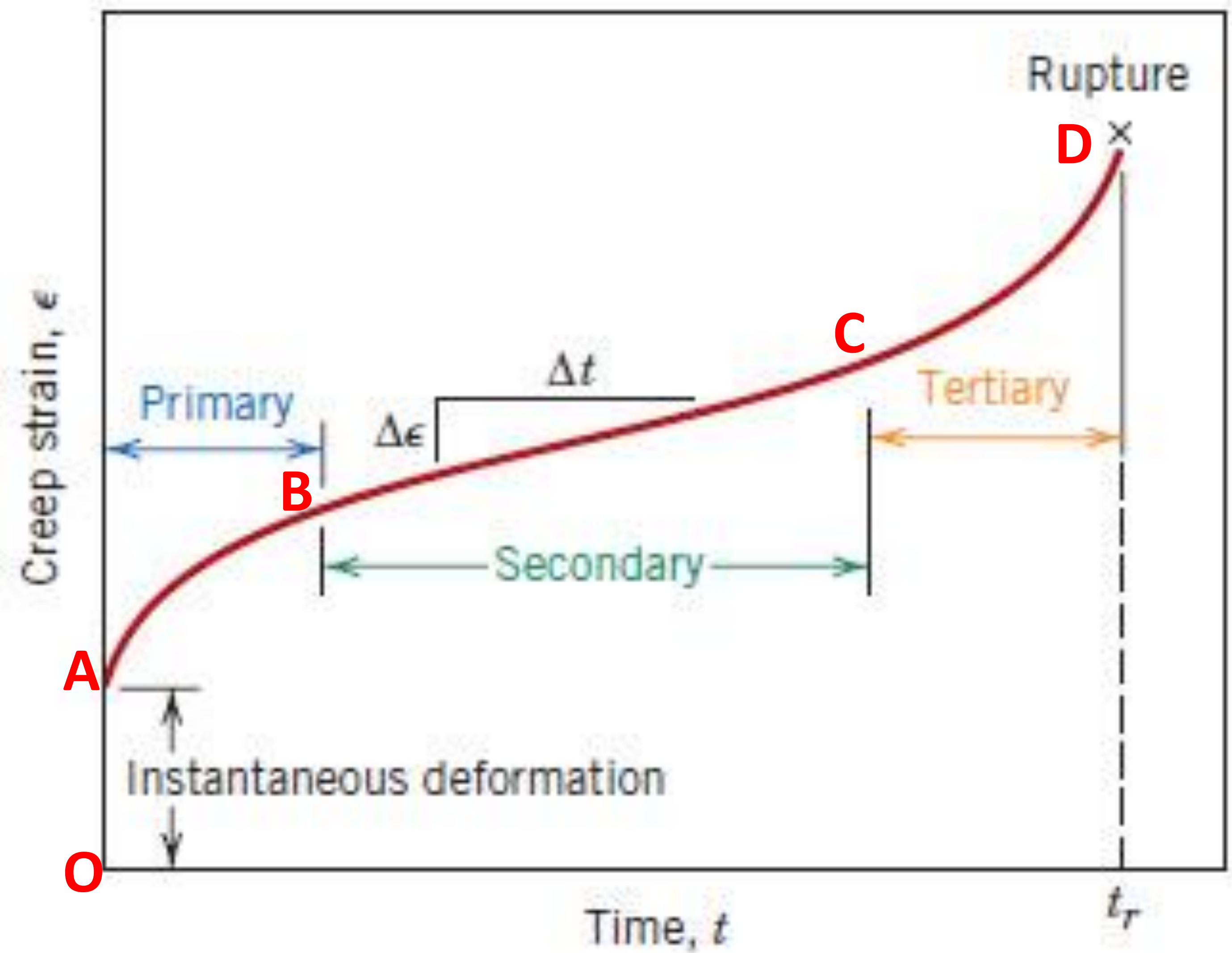


Creep

Creep Strength of the material is defined as the maximum stress that the material can withstand for a specified length of time without excessive deformation

Creep rupture strength of the material is the maximum stress that the material can withstand for a specified length of time without rupture

Creep



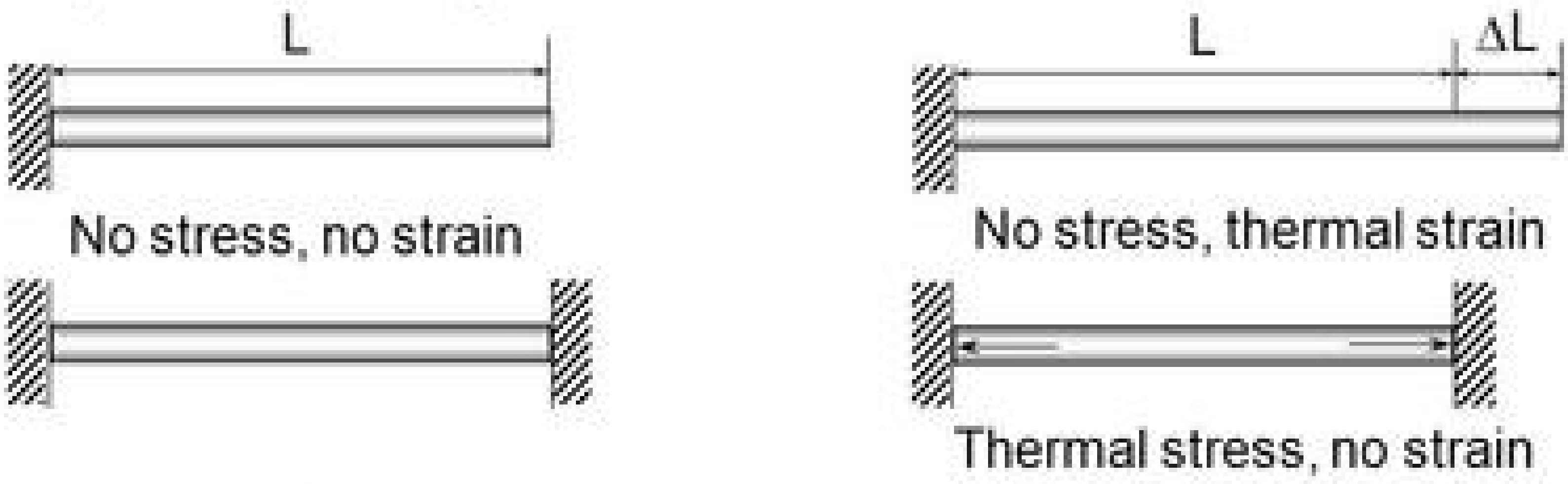
AB: Strain hardens

BC: highly mobile dislocations counteract the strain hardening

CD: formation of voids along grain boundaries

Thermal Stresses

Stress that arise due to the variation in temperature



For a rod, $$\sigma = -\alpha E \Delta T$$

For a plate (2D) $$\sigma_x = \sigma_y = \frac{-\alpha E \Delta T}{1 - \mu}$$

For a box (3D) $$\sigma_x = \sigma_y = \sigma_z = \frac{-\alpha E \Delta T}{1 - 2\mu}$$